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Deep Free Trade Agreements with the Harmonization of Production Subsidies

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Abstract: This study examines the welfare effect of a deep free trade agreement (FTA) with the harmonization of production subsidies in an asymmetric two-country model of Cournot oligopoly, where a large country has a larger domestic market and more firms than a small country. The main finding is that a deep FTA will decrease the welfare of the small country if the number of firms in the large country is sufficiently small and the difference in market size between the two countries is extremely large. It will reduce the welfare of the large country if the number of these firms is somewhat great and the market size gap is not too large. This sharply contrasts with results of the traditional model with tariffs alone.

Keywords: Free trade agreement; production subsidies; tariffs; Cournot oligopoly

JEL Classification Numbers: F12; F13; F15

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1. Introduction

There has been a new trend of forming "deep" free trade agreements (FTAs), covering not just the elimination of import tariffs, but also a broad range of other areas such as the harmonization of domestic systems. For example, in May 2021, the United Kingdom and the European Union entered into the Trade and Cooperation Agreement, a deep FTA, including matters such as competition and subsidies, to ensure a level playing field.

In the existing theoretical studies on FTAs, Kose and Riezman (2000) and Bond et al. (2004) use a three-country endowment model to investigate the welfare effects of FTAs.[1] Yi (2000), Ornelas (2005), Saggi (2006), and Nomura et al. (2013) employ a multi-country Cournot oligopoly model to analyze the effects of FTAs on welfare, and the incentives for multilateral trade liberalization of member and non-member countries.[2] However, these analyses do not consider deep FTAs involving the elimination of import tariffs, and the harmonization of domestic policies. In contrast, using a three-country Cournot oligopoly model with endogenously determined tariffs and standards, Kawabata and Takarada (2021) examine the effect of deep FTAs with the harmonization of standards on the multilateral harmonization of standards, as well as multilateral free trade.[3]

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[1] Saggi and Yildiz (2010) explore the relationship between FTAs and multilateral trade liberalization in a three-country endowment model.

[2] Kawabata and Takarada (2015) and Yanase et al. (2012) examine the welfare effect of FTAs in a three-country Bertrand oligopoly model with product differentiation, and a three-country Cournot oligopoly model with a vertical industry structure, respectively. Ornelas (2007) investigates the welfare effect of customs unions in a three-country Cournot oligopoly model.

[3] Limão (2007) develops a model of preferential trade agreements with cooperation in non-trade issues to analyze its implications for global free trade and welfare.

The purpose of this study is to explore the welfare effect of a deep FTA with the harmonization of production subsidies. We construct an asymmetric two-country model of Cournot oligopoly, in which a "large country" has a larger domestic market and more firms than a "small country."[4] In the pre-FTA regime, each country's government sets its import tariff and product subsidy to maximize individual welfare. When the two countries form a deep FTA, their governments eliminate tariffs on each other, and set a common product subsidy to maximize their joint welfare.

With regard to the effect of a deep FTA on production subsidies, we obtain the following outcome: After the formation of a deep FTA with the harmonization of production subsidies, member countries will, in many cases, raise their production subsidies to correct the domestic distortion due to imperfect competition. However, one member country that provides extremely high production subsidies under the pre-FTA regime will reduce its production subsidies, which may decrease its exports.

With regard to the welfare effect of a deep FTA, the following findings are obtained:

(i) The greater the number of firms in the large country, and/or the smaller the difference in market size between the small and large countries, higher the chances of a deep FTA with the harmonization of product subsidies of increasing the welfare of the small country.

[4] Nomura et al. (2013) and Chang and Xiao (2015) assume asymmetry in market size between FTA member countries, but do not consider asymmetric number of firms. The three-country model complicates the analysis when taking into account the differences in both the market size and the number of firms among countries. Therefore, we use a two-country model to focus on the impact of a deep FTA on member countries.

However, if the number of these firms is sufficiently small, and the market size gap is extremely large, it will reduce the small country's welfare through a significant increase in the costs of production subsidies.

(ii) The smaller the number of firms in the large country, and/or the larger the degree of market size asymmetry, the more likely it is that a deep FTA with harmonization of production subsidies will improve the welfare of the large country. However, if the number of these firms is somewhat large, and the market size differential is not too large, it will worsen the large country's welfare because it will not increase the consumer surplus too much.

This result sharply contrasts with that of the traditional model with import tariffs alone, as shown in Appendix B. The addition of changes in production subsidies due to deep FTAs creates a difference in the results.

The remainder of this paper is organized as follows. Section 2 describes the model and derives the pre-FTA equilibrium. Section 3 analyzes the effects of a deep FTA on production subsidies, trade volume, and welfare. Section 4 concludes.

2. The Model

2.1 Basic settings

There are two countries: a "large country" (L) and a "small country" (S). Country L, with a larger domestic market, has $n(\geq 1)$ identical firms, and country S has one firm. All firms have the same marginal cost c, and engage in Cournot competition in each of the national markets, which are

assumed to be segmented. The price p_i in country i $(i = S, L)$ is determined by the inverse demand function:

$$p_i(Q_i) = \alpha - \beta_i Q_i, \quad \alpha > 0 \tag{1}$$

where $Q_i = q_{iS} + nq_{iL}$ is the total quantity supplied to country i, and q_{ij} is the quantity supplied by country j's firm (firm j) to country i $(i, j = S, L)$.[5] Following Nomura et al. (2013), we assume that $\beta_S = 1 > \beta_L = \beta > 0$. The smaller the value of β, the larger the market size of country L. The government in country i provides a specific production subsidy s_i, and imposes a specific import tariff t_i.

The profits of firm i $(i = S, L)$ are given by

$$\pi_i = (p_i - c + s_i)q_{ii} + (p_j - c + s_i - t_j)q_{ji}, \quad i, j = S, L, \quad i \neq j \tag{2}$$

The first and second terms in Equation (2) are firm i's local and export profits, respectively.

The welfare of country i $(i = S, L)$ is calculated as its consumer surplus, plus firm i's profits, plus its tariff revenue, minus production subsidy costs:

$$W_S = CS_S + \pi_S + t_S n q_{SL} - s_S \sum_{i=S,L} q_{iS} \tag{3}$$

$$W_L = CS_L + n\pi_L + t_L q_{LS} - n s_L \sum_{i=S,L} q_{iL} \tag{4}$$

where consumer surplus is denoted by $CS_i = \int_0^{Q_i} p_i(x)dx - p_i(Q_i)Q_i$.

The model involves two stages of decision making. In Stage 1, each government endogenously determines its own production subsidy and tariff. In Stage 2, firms decide their outputs. We then derive the subgame perfect Nash equilibrium of this model.

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[5] It is assumed that $\alpha > c$.

2.2 Market equilibrium

Using backward induction, we begin with Stage 2. Each firm chooses its output to maximize its own profits, taking the outputs of the rivals, and the production subsidy and tariff of each country as given. From Equation (2), firm i's and j's first-order conditions for profit maximization are

$$p_i - c + s_i + q_{ii}p_i' = 0,$$

$$p_i - c + s_j - t_i + q_{ij}p_i' = 0, \;\; i,j = S,L, \;\; i \neq j \tag{5}$$

By solving these conditions simultaneously, we obtain firm *S*'s and firm *L*'s Cournot-Nash equilibrium sales in country *S*'s and *L*'s markets as follows:

$$q_{SS} = \frac{\alpha - c + (n+1)s_S + nt_S - ns_L}{n+2}, \quad q_{SL} = \frac{\alpha - c - s_S - 2t_S + 2s_L}{n+2},$$
$$q_{LS} = \frac{\alpha - c - ns_L - (n+1)t_L + (n+1)s_S}{\beta(n+2)}, \quad q_{LL} = \frac{\alpha - c + 2s_L + t_L - s_S}{\beta(n+2)} \tag{6}$$

From Equation (6), the equilibrium total sales (consumption) in countries *S* and *L* are given by

$$Q_S = \frac{(n+1)(\alpha - c) + s_S - nt_S + ns_L}{n+2}, \quad Q_L = \frac{(n+1)(\alpha - c) + ns_L - t_L + s_S}{\beta(n+2)} \tag{7}$$

2.3 Pre-FTA regime

We now turn to the endogenous determination of production subsidies and import tariffs in Stage 1. In the pre-FTA regime, the two countries independently, and simultaneously choose their respective production subsidies and tariffs.

The government of country i simultaneously determines its production subsidy s_i, and its

tariff t_i to maximize its own welfare, taking the other country's production subsidy and tariff as given.[6] Country *S*'s first-order conditions for welfare maximization are given as follows:[7]

$$\frac{\partial W_S}{\partial s_S} = Q_S \frac{\partial Q_S}{\partial s_S} + \left(2q_{SS}\frac{\partial q_{SS}}{\partial s_S} + 2\beta q_{LS}\frac{\partial q_{LS}}{\partial s_S}\right) + t_S n \frac{\partial q_{SL}}{\partial s_S} - \left[q_{SS} + q_{LS} + s_S\left(\frac{\partial q_{SS}}{\partial s_S} + \frac{\partial q_{LS}}{\partial s_S}\right)\right] = 0, \quad (8a)$$

$$\frac{\partial W_S}{\partial t_S} = Q_S \frac{\partial Q_S}{\partial t_S} + 2q_{SS}\frac{\partial q_{SS}}{\partial t_S} + \left(nq_{SL} + t_S n\frac{\partial q_{SL}}{\partial t_S}\right) - s_S\frac{\partial q_{SS}}{\partial t_S} = 0 \quad (8b)$$

The first term in Equations (8a) and (8b) is the effect of the production subsidy s_S, and tariff t_S on consumer surplus, the second term is the effect on firm *S*'s profits, the third term is the change in tariff revenues, and the final term is the change in production subsidy costs.[8]

By substituting Equations (6) and (7) into Equations (8a) and (8b), and rearranging them, we obtain country *S*'s reaction functions:

$$s_S = \frac{[(2n+1)\beta + n](\alpha - c) - 3n\beta t_S - n[(n-1)\beta + n]s_L - n(n+1)t_L}{(2n+1)\beta + 2(n+1)}, \quad (9a)$$

$$t_S = \frac{3(\alpha - c) - 3s_S - (n-4)s_L}{n+8} \quad (9b)$$

Country *L*'s first-order conditions are as follows:[9]

$$\frac{\partial W_L}{\partial s_L} = \beta Q_L \frac{\partial Q_L}{\partial s_L} + n\left(2q_{SL}\frac{\partial q_{SL}}{\partial s_L} + 2\beta q_{LS}\frac{\partial q_{LS}}{\partial s_S}\right) + t_L\frac{\partial q_{LS}}{\partial s_L} - n\left[q_{SL} + q_{LL} + s_L\left(\frac{\partial q_{SL}}{\partial s_L} + \frac{\partial q_{LL}}{\partial s_L}\right)\right] = 0, \quad (10a)$$

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[6] Lai and Hu (2008) assume that import tariffs are determined before domestic regulations. Even if each country's government sets its tariff in the first stage, and its production subsidy in the second stage, our main findings do no change qualitatively.

[7] The second-order conditions are satisfied: $\partial^2 W_S/\partial s_S^2 = -[(2n+1)\beta + 2(n+1)]/\beta(n+2)^2 < 0$, $\partial^2 W_S/\partial t_S^2 = -n(n+8)/(n+2)^2 < 0$, and $(\partial^2 W_S/\partial s_S^2)(\partial^2 W_S/\partial t_S^2) - (\partial^2 W_S/\partial s_S \partial t_S)^2 = 2n[(n+2)^2\beta + (n+8)(n+1)]/\beta(n+2)^4 > 0$.

[8] Country *S*'s and *L*'s consumer surplus are given by $(Q_S)^2/2$ and $\beta(Q_L)^2/2$, respectively. Using the first-order conditions (5), firm i's profits are given by $\pi_i = (q_{Si})^2 + \beta(q_{Li})^2$ $(i = S, L)$.

[9] The second-order conditions are satisfied: $\partial^2 W_L/\partial s_L^2 = -n^2(4\beta + 3)/\beta(n+2)^2 < 0$, $\partial^2 W_L/\partial t_L^2 = -(2n^2 + 4n + 3)/\beta(n+2)^2 < 0$, and $(\partial^2 W_L/\partial s_L^2)(\partial^2 W_L/\partial t_L^2) - (\partial^2 W_L/\partial s_L \partial t_L)^2 = 2n^2[2\beta(2n^2 + 4n + 3) + (n+2)^2]/\beta^2(n+2)^4 > 0$.

$$\frac{\partial W_L}{\partial t_L} = \beta Q_L \frac{\partial Q_L}{\partial t_L} + 2n\beta q_{LL} \frac{\partial q_{LL}}{\partial t_L} + \left(q_{LS} + t_L \frac{\partial q_{LS}}{\partial t_L}\right) - n s_L \frac{\partial q_{LL}}{\partial t_L} = 0 \qquad (10b)$$

The first term in Equations (10a) and (10b) is the effect of s_L and t_L on consumer surplus, the second term is the effect on firm L's profits, the third term is the tariff revenue effect, and the final term is the effect on production subsidy spending.

By substituting Equations (6) and (7) into Equations (10a) and (10b), and rearranging them, we obtain country L's reaction functions:

$$s_L = \frac{[3-(n-2)\beta](\alpha - c) - (2n+1)t_L + [(n-2)\beta + n - 1]s_S + 2\beta(n-2)t_S}{n(4\beta+3)}, \quad (11a)$$

$$t_L = \frac{(2n+1)(\alpha - c) - n(2n+1)s_L + (n^2+n+1)s_S}{2n^2+4n+3} \qquad (11b)$$

Note that Equations (9a), (9b), (11a), and (11b) indicate that the production subsidy and tariff of one country depend on the other country's policy, which is in contrast to the model with only import tariffs ($s_S = s_L = 0$), where the tariff in one country does not depend on the tariff of the other country in the case of market segmentation, and constant marginal cost.

By solving Equations (9a), (9b), (11a), and (11b), we obtain the Nash equilibrium production subsidies, and import tariffs under the pre-FTA regime:[10]

$$s_S^* = \frac{\beta}{F}[(2n^2+4n+3)(n+2)^2\beta + n(3n^2+12n+16)](\alpha - c), \qquad (12)$$

$$s_L^* = \frac{1}{F}[(14+15n+5n^2-2n^3-n^4)\beta + (n+8)(n+2)^2](\alpha - c), \qquad (13)$$

$$t_S^* = \frac{1}{F}[(n^4+8n^3+22n^2+27n+16)\beta + 2(n+2)^3](\alpha - c), \qquad (14)$$

$$t_L^* = \frac{\beta}{F}[(n+1)(n+2)^3\beta + n^4+8n^3+25n^2+32n+8](\alpha - c), \qquad (15)$$

[10] See Appendix A for the pre-FTA and FTA sales of firms *S* and *L,* and the pre-FTA and FTA welfare levels of countries *S* and *L*.

where $F \equiv (2n^2+4n+3)(n+2)^2\beta^2 + 6(n^4+6n^3+13n^2+13n+4)\beta + n(n+8)(n+2)^2$. Equations (12), (14), and (15) indicate that country *S*'s production subsidy and the tariff of both the countries are always positive. Equation (13) indicates that country *L*'s production subsidy is positive when the number of firms L is less than or equal to four ($n \leq 4$). Country *L*'s production subsidy is more likely to be negative (production tax) if n is sufficiently large, and the market size of country *L* is not too large in comparison with that of country *S* (β is not too small).[11]

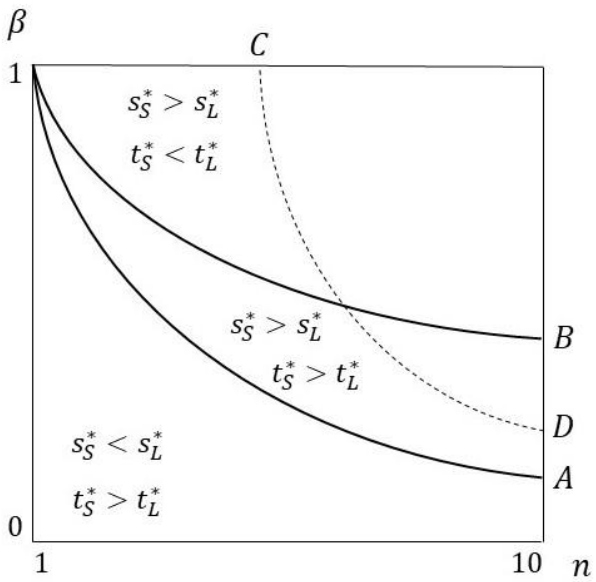

Figure 1. Comparison of country *S*'s and *L*'s pre-FTA policies
Notes: $A = (10, 0.12839)$, $B = (10, 0.4175)$, $C = (4.9302, 1)$, $D = (10, 0.22865)$

Figure 1 illustrates the comparison between country *S*'s and *L*'s policies under the pre-FTA regime. Country *S*'s production subsidy is larger (smaller) than country *L*'s production subsidy in

[11] In Figure 1, $s_L^* < 0$ ($s_L^* > 0$) in the area above (below) the dashed curve *CD*.

the area above (below) curve $1A$, *that is*, $s_S^* > s_L^*$ ($s_S^* < s_L^*$). Country *S*'s tariff is lower (higher) than country *L*'s tariff in the area above (below) curve $1B$, *that is*, $t_S^* < t_L^*$ ($t_S^* > t_L^*$).

The smaller the number of firms L, n, and/or the larger the market size of country L in comparison with country *S* (the smaller the value of β), the more likely it is that country *L*'s production subsidy will exceed country *S*'s production subsidy.[12] In addition, the smaller the value of n, and/or the larger the market size of country *L* (the smaller the value of β), it is more likely that country *S*'s tariff will exceed country *L*'s tariff.[13] The intuition behind this result is as follows: Country i's government uses a production subsidy to reduce domestic distortion due to imperfect competition, and to shift profits from firm j to firm i in the export market. It uses an import tariff to extract profits from firm j in the domestic market ($i, j = S, L,\ i \neq j$). If n is small enough, and β is quite small, the domestic distortion in country *L* is significantly large, which strengthens the incentive for country *L* to subsidize production. Country *L*'s higher production subsidy increases the export profits earned by firm *L* in country *S*'s market that country *S* can extract with its tariff, thereby raising country *S*'s tariff.

With regard to the difference in production subsidies, we have the following lemma:

Lemma 1. *(i) Country L's production subsidy is much higher than country S's production subsidy*

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[12] As n decreases, the range of β for which $s_S^* < s_L^*$ increases. If $n = 8$, $s_S^* < s_L^*$ for $0 < \beta < 0.1638$. If $n = 2$, $s_S^* < s_L^*$ for $0 < \beta < 0.61666$. As β becomes smaller, $s_S^* < s_L^*$ holds for a wider range of n. If $\beta = 0.8$, $s_S^* < s_L^*$ for $n = 1$. If $\beta = 0.2$, $s_S^* < s_L^*$ for $1 \leq n \leq 6$.

[13] This contrasts with the result in the model with only import tariffs, where country *S*'s tariff is always higher than country *L*'s tariff, *that is*, $t_S^* \geq t_L^*$. For a point of comparison, we deal with the model with only tariffs in Appendix B.

if the number of firms L, n, is sufficiently small, and the market size of country L is extremely large (β is too small).

(ii) Country S's production subsidy is substantially larger than country L's production subsidy if n is quite large, and the market size of country L is slightly larger than that of country S.

3. Deep FTA with the harmonization of production subsidies

3.1 Effect of a deep FTA on production subsidies

Next, we consider a deep FTA with the harmonization of production subsidies. When countries *S* and *L* form a deep FTA, the governments of the FTA member countries eliminate tariffs on each other, $t_S = t_L = 0$, and set a common production subsidy, $s_S = s_L = s_F$, to maximize their joint welfare.[14]

By substituting $t_S = t_L = 0$ into Equations (3) and (4), and maximizing the aggregate welfare of the FTA members, $W_S + W_L$, with respect to s_F, we obtain the common production subsidy under a deep FTA.[15]

$$s_F = \frac{\alpha - c}{n+1} \tag{16}$$

From Equation (16), the common production subsidy is always positive.

Figure 2 shows a comparison between the pre-FTA and the FTA production subsidies. The

[14] See Appendix C for the case of FTA without the harmonization of production subsidies.

[15] The first-order condition for the joint-welfare maximization is given by $d(W_S + W_L)/ds_F = (\beta+1)(n+1)[\alpha - c - (n+1)s_F]/\beta(n+2)^2 = 0$. The second-order condition is satisfied: $d^2(W_S + W_L)/ds_F^2 = -(\beta+1)(n+1)^2/\beta(n+2)^2 < 0$.

common production subsidy under a deep FTA is larger (smaller) than country *S*'s pre-FTA production subsidy in the region below (above) curve EF, *that is*, $s_F > s_S^*$ ($s_F < s_S^*$). The FTA common production subsidy is higher (lower) than country *L*'s pre-FTA production subsidy in the region above (below) curve GH, *that is*, $s_F > s_L^*$ ($s_F < s_L^*$).

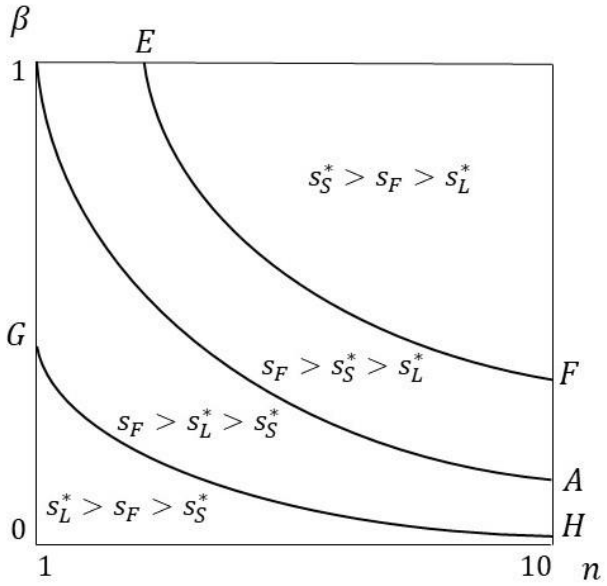

Figure 2. Comparison of pre-FTA and FTA production subsidies
Notes: $A = (10, 0.12839)$, $E = (2.8053, 1)$, $F = (10, 0.36488)$, $G = (1, 0.41786)$, $H = (10, 0.011285)$

With regard to the effect of a deep FTA on the production subsidies of countries *L* and *S*, we obtain the following proposition.

Proposition 1. *(i) After the formation of a deep FTA between countries S and L, country L will reduce its production subsidy if the number of firms L, n, is sufficiently small, and the market size of country L is sufficiently large (β is much less than 0.41786). Otherwise, it will raise its*

production subsidy.

(ii) Country S will decrease its production subsidy if $n \geq 3$*, and the market size of country L is not much larger than that of country S (*β *is not too small). Otherwise, it will increase its production subsidy.*

We can explain the intuition behind this proposition as follows. In the area below curve EF, and above curve GH in Figure 2, countries *L* and *S* provide a higher production subsidy under the deep FTA than under the pre-FTA regime to correct the domestic distortion so that the prices are equal to the marginal cost in both countries' markets.[16] However, from Lemma 1, if the number of firms *L*, n, is sufficiently small, and the market size of country *L* is extremely large (the area below curve GH), country *L*'s pre-FTA production subsidy would be too high. In this case, country *L* will lower its production subsidy after the formation of the FTA. In addition, if $n \geq 3$, and the market size of country *L* is somewhat larger than that of country *S* (the area above curve EF), country *S*'s pre-FTA production subsidy will be significantly large, and therefore, country *S* will reduce its production subsidy.

Note that when the values of n and β are sufficiently small, an increase in s_S due to the deep FTA is sufficiently large.

[16] Appendix A shows that the common production subsidy under the FTA makes prices in countries *S* and *L* equal to the marginal cost, and hence, eliminates the domestic distortion.

3.2 Effect of a deep FTA on trade

Next, we examine the effect of a deep FTA with the harmonization of production subsidies on the equilibrium trade volume.

From Equations (A1) and (A4) in Appendix A, the deep FTA between countries *S* and *L* increases firm *S*'s exports to country *L* ($q_{LS}^F > q_{LS}^*$), if the number of firms *L* is six or less ($n \leq 6$). However, it may decrease firm *S*'s exports ($q_{LS}^F < q_{LS}^*$), if n is large enough, and the market size of country *L* is slightly larger than that of country *S* (β is close to 1). This is because, despite the elimination of country *L*'s import tariff, country *S*'s production subsidy greatly decreases after the deep FTA.

The deep FTA expands firm *L*'s exports to country *S* ($q_{SL}^F > q_{SL}^*$), if $n \geq 2$. However, it may reduce firm *L*'s exports, if $n = 1$, and the market size of country *L* is extremely large ($\beta < 0.057452$). The reason for this is that although country *S* removes its import tariff, country *L* sharply reduces its production subsidy with a deep FTA.

The above finding is in contrast to the result of the model with only import tariffs where the FTAs always increase the trade flows between member countries (see Appendix B). In the presence of production subsidies, changes in subsidies due to deep FTAs may cause a reduction in trade volume between member countries.

In addition, the deep FTA, unambiguously, increases consumption in member countries ($Q_i^F > Q_i^*,\ i = S, L$).[17]

[17] See Appendix A for the effects of a deep FTA on the domestic sales of firms *S* and *L*.

3.3 Effect of a deep FTA on welfare

Finally, we investigate the effect of a deep FTA with the harmonization of production subsidies on the welfare levels of the member countries.

Figure 3 illustrates the comparison between pre-FTA and FTA welfare levels. Under the deep FTA, country *S* is better (worse) off in the area above (below) curve IJ, *that is*, $W_S^F > W_S^*$ ($W_S^F < W_S^*$). The deep FTA makes country *L* better (worse) off in the area below (above) curve KL, *that is*, $W_L^F > W_L^*$ ($W_L^F < W_L^*$).

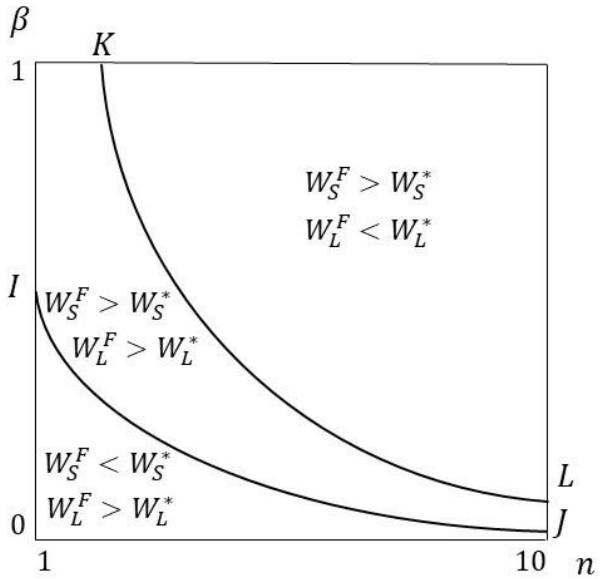

Figure 3. Comparison of pre-FTA and FTA welfare levels

Notes: $I = (1, 0.51697)$, $J = (10, 0.018755)$, $K = (2.1184, 1)$, $L = (10, 0.088699)$

With regard to the effect of a deep FTA on country *S*'s and *L*'s welfare levels, we obtain the

following proposition.

Proposition 2. *(i) The greater the number of firms L, n, and/or the closer the market size of country L to that of country S (the closer β is to 1), higher the chances of country S's welfare increasing under the deep FTA. However, it will decrease country S's welfare if n is sufficiently small, and the market size of country L is extremely large (β is even less than 0.516697).*

(ii) The smaller the number of firms L, and/or the larger the market size of country L (the smaller the value of β), the more likely it is that the deep FTA will improve country L's welfare. However, it will worsen country L's welfare if $n \geq 3$, and the market size of country L is not too large in comparison with country S (β is not too small).

We can explain the intuition behind the above proposition in the following manner.

(i) For country S's welfare

Suppose that the number of firms L, n, is sufficiently small, and the market size of country L is too large (β is extremely small). In that case, on the one hand, country S's consumer surplus, and firm S's profits increase with the formation of the deep FTA between countries S and L.[18] This is a positive effect of the FTA. On the other hand, country S's tariff revenue disappears, and the costs of production subsidy increase because of rising subsidies. This is a negative effect. For sufficiently small n and β values, following the FTA, country S's tariff falls sharply, and its production subsidy rises significantly, thereby strengthening the negative effect (see the

[18] If the values of n and β are large, the deep FTA will reduce firm S's profits.

discussion in Sections 2.3 and 3.1). Therefore, the negative effect outweighs the positive effect, so the deep FTA worsens country *S*'s welfare. However, the larger the values of n and/or β, the more likely it is that the positive effect will become dominant, and hence, the deep FTA will improve country *S*'s welfare.

(ii) For country L's welfare

Suppose that the number of firms L, n, is somewhat large, and the market size of country L is slightly larger than that of country S (β is close to 1). Then, after the deep FTA is formed, country L's tariff revenue disappears, and its production subsidy spending increases owing to an increase in the subsidies. This is a negative effect of the FTA. On the other hand, country L's consumer surplus, and firm L's profits increase.[19] This is a positive effect. For high n and β values, the increase in country L's consumer surplus is quite small, thereby weakening the positive effect. Thus, the negative effect dominates, and the deep FTA hurts country L. As the values of n and/or β decrease, the positive effect becomes dominant, and consequently, the deep FTA benefits country L.

This finding sharply contrasts with the result of the traditional model with only import tariffs ($s_S = s_L = 0$), given as Proposition A1 in Appendix B.[20] Comparing Figures 3 and A1 in Appendix B, in the region below curve IJ, where a deep FTA with the harmonization of

[19] If the values of n and β are small, firm L's profits decrease with the deep FTA.
[20] In the case of symmetric countries ($n = \beta = 1$), a deep FTA with production subsidies increases members' welfare, and this is in line with the result in the model with only tariffs (see Figures 3 and A1).

production subsidies reduces country S's welfare, an FTA increases country S's welfare in the model with only tariffs. In the region above curve MN, drawn in Figure A1, where an FTA makes country S worse off in the model without production subsidies, a deep FTA with production subsidies makes country S better off relative to the pre-FTA regime. If the number of firms L is sufficiently small, and the market size of country L is significantly large (the value of β is small enough), a deep FTA with production subsidies improves country L's welfare, whereas an FTA worsens country L's welfare in the model with only tariffs. If the values of n and β are quite large, country L gains from an FTA in the absence of production subsidies; however, it suffers losses from a deep FTA in its presence.[21] The addition of changes in production subsidies due to deep FTAs gives the opposite result to that obtained in the model with only tariffs.

4. Conclusion

We investigated the welfare effect of a deep FTA with the harmonization of production subsidies in an asymmetric two-country model of Cournot oligopoly. A deep FTA is formed between the small country and the large country with a larger domestic market and more firms. Our contribution is to clarify how deep FTAs covering the elimination of import tariffs, and the harmonization of production subsidies modify the result of traditional frameworks with only

[21] Focusing on the difference in the market size, when the market size gap between countries S and L is sufficiently large, an FTA increases country S's welfare, and decreases country L's welfare in the model with only tariffs. In contrast, a deep FTA with production subsidies is likely to benefit country L, and hurt country S in a similar scenario.

tariffs.

The main conclusion is that a deep FTA with the harmonization of production subsidies is more likely to improve the welfare of the small (large) country, the greater (smaller) the number of firms in the large country, and/or the smaller (larger) the difference in market size between the small and large countries. However, it will worsen the small (large) country's welfare if the number of these firms is sufficiently small (somewhat great), and the market size gap is extremely large (not too large). This is in sharp contrast to the result of the traditional model with tariffs alone, as shown in Appendix B.

Several directions for future research appear worthwhile. It would be important to extend this to a multi-country model, and examine whether a deep FTA with production subsidies facilitates or hinders the achievement of multilateral trade liberalization with the harmonization of production subsidies. It would also be interesting to incorporate research and development (R&D) investment by firms to reduce costs, and deal with R&D subsidies.

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Appendix A. Pre-FTA and FTA sales and welfare

Substituting Equations (12) – (15) into Equations (6) and (7) yields firm *S*'s and *L*'s equilibrium sales and total sales (consumption) in country *S*'s and *L*'s markets under the pre-FTA regime:

$$q_{SS}^* = \frac{\alpha - c}{F}[(2n^2 + 4n + 3)(n + 2)^2\beta^2 + (2n^4 + 15n^3 + 38n^2 + 42n + 12)\beta + 2n(n + 2)^2],$$

$$q_{SL}^* = \frac{\alpha - c}{F}[(2n + 5)(n^2 + 2n + 2)\beta + (n + 4)(n + 2)^2],$$

$$q_{LS}^* = \frac{\alpha - c}{F}[(n^2 + 2n + 2)(n + 2)^2\beta + 2n^3 + 9n^2 + 16n + 8],$$

$$q_{LL}^* = \frac{\alpha - c}{\beta F}[(n + 1)(n + 2)^2\beta^2 + (5n^3 + 27n^2 + 47n + 30)\beta + (n + 8)(n + 2)^2],$$

$$Q_S^* = \frac{\alpha - c}{F}[(2n^2 + 4n + 3)(n + 2)^2\beta^2 + 4(n + 3)(n^3 + 3n^2 + 4n + 1)\beta + n(n + 6)(n + 2)^2],$$

$$Q_L^* = \frac{\alpha - c}{\beta F}[(2n^2 + 3n + 2)(n + 2)^2\beta^2 + (5n^4 + 29n^3 + 56n^2 + 46n + 8)\beta$$

$$+n(n + 8)(n + 2)^2] \qquad (A1)$$

Using Equations (12) – (15) and (A1) in Equations (3) and (4), we obtain the pre-FTA welfare levels of countries *S* and *L*:

$$W_S^* = \frac{(\alpha - c)^2}{2F^2}[(2n^2 + 4n + 3)^2(n + 2)^4\beta^4$$

$$+2(11n^6 + 88n^5 + 291n^4 + 526n^3 + 540n^2 + 298n + 64)(n + 2)^2\beta^3$$

$$+2(16n^8+208n^7+1147n^6+3532n^5+6650n^4+7774n^3+5414n^2+1952n+248)\beta^2$$

$$+2(5n^8+92n^7+636n^6+2264n^5+4621n^4+5508n^3+3584n^2+1024n+64)\beta$$

$$+n(n+8)(n^2+8n+4)(n+2)^4], \quad (A2)$$

$$W_L^* = \frac{(\alpha-c)^2}{2\beta F^2}[(3n^2+6n+4)(2n^2+4n+3)(n+2)^4\beta^4$$

$$+2(14n^8+163n^7+826n^6+2395n^5+4342n^4+5000n^3+3522n^2+1344n+192)\beta^3$$

$$+(41n^8+514n^7+2725n^6+8024n^5+14318n^4+15584n^3+9700n^2+2784n+192)\beta^2$$

$$+4n(3n^5+42n^4+181n^3+335n^2+284n+64)(n+2)^2\beta+n^2(n+8)^2(n+2)^4] \quad (A3)$$

Substituting Equation (16) into Equations (6) and (7) yields firm *S*'s and *L*'s equilibrium sales and total sales (consumption) in country *S*'s and *L*'s markets under the deep FTA:

$$q_{SS}^F = q_{SL}^F = \frac{\alpha-c}{n+1}, \quad q_{LS}^F = q_{LL}^F = \frac{\alpha-c}{\beta(n+1)},$$
$$Q_S^F = \alpha - c, \quad Q_L^F = \frac{\alpha-c}{\beta} \quad (A4)$$

From (A4), the FTA equilibrium prices in countries *S* and *L* are equal to the marginal cost: $p_S^F = p_L^F = c$.

Using Equations (16) and (A4) in Equations (3), (4), we obtain the FTA welfare levels of countries *S* and *L*:

$$W_S^F = \frac{(\alpha-c)^2}{2}, \quad (A5)$$
$$W_L^F = \frac{(\alpha-c)^2}{2\beta} \quad (A6)$$

Comparing Equations (A1) and (A4), the deep FTA between countries *S* and *L* reduces the domestic sales of firm *S* ($q_{SS}^F < q_{SS}^*$), if the number of firms *L*, n is six or more ($n \geq 6$). However, if the value of n is small, and the market size of country *L* is sufficiently large (β is small enough),

it may increase firm *S*'s domestic sales ($q_{SS}^F > q_{SS}^*$) because, in spite of the elimination of country *S*'s tariff, country *S*'s production subsidy greatly increases with the FTA.

After the deep FTA, country *L*'s domestic sales are more likely to decrease ($q_{LL}^F < q_{LL}^*$), if the value of n is smaller, and/or the market size of country *L* is larger (the value of β is smaller). However, if the values of n and β are large, domestic sales may increase ($q_{LL}^F > q_{LL}^*$) because country *L*'s production subsidy rises significantly, even though country *L*'s tariff is removed.

Appendix B. Model with only import tariffs

Suppose that countries *S* and *L* use only import tariffs ($s_S = s_L = 0$), as in traditional FTA frameworks. By substituting $s_S = s_L = 0$ into Equations (6) and (7), the market equilibrium is derived.

Under the pre-FTA regime, the government of country i determines its tariff t_i to maximize its own welfare, taking the other country's tariff as given. Country *S*'s first-order condition for welfare maximization is given by

$$\frac{\partial W_S}{\partial t_S} = \frac{n}{(n+2)^2}[3(\alpha - c) - (n+8)t_S] = 0 \qquad (B1)$$

From Equation (B1), the pre-FTA tariff of country *S* is

$$t_S^* = \frac{3(\alpha - c)}{n+8} \qquad (B2)$$

Country *L*'s first-order condition for welfare maximization is

$$\frac{\partial W_L}{\partial t_L} = \frac{1}{\beta(n+2)^2}[(2n+1)(\alpha - c) - (2n^2 + 4n + 3)t_L] = 0 \qquad (B3)$$

From Equation (B3), the pre-FTA tariff of country *L* is

$$t_L^* = \frac{(2n+1)(\alpha - c)}{2n^2 + 4n + 3} \tag{B4}$$

We can show that $t_S^* \geq t_L^*$ ($t_S^* = t_L^*$ only when $n = 1$). Country *S*'s tariff is higher than country *L*'s tariff if the number of firm *L* is two or more ($n \geq 2$). Note that Equations (B1) and (B3) indicate that the optimal tariff in one country does not depend on the other country's tariff in the model with only tariffs (the strategic independence of the countries' tariff policies).

Using Equations (B2) and (B4) yields the pre-FTA equilibrium sales in country *S*'s and *L*'s markets:

$$\begin{aligned} q_{SS}^* &= \frac{4(\alpha - c)}{n+8}, \quad q_{SL}^* = \frac{\alpha - c}{n+8}, \\ q_{LS}^* &= \frac{\alpha - c}{\beta(2n^2 + 4n + 3)}, \quad q_{LL}^* = \frac{2(n+1)(\alpha - c)}{\beta(2n^2 + 4n + 3)}, \\ Q_S^* &= \frac{(n+4)(\alpha - c)}{n+8}, \quad Q_L^* = \frac{(2n^2 + 2n + 1)(\alpha - c)}{\beta(2n^2 + 4n + 3)} \end{aligned} \tag{B5}$$

Using Equations (B2), (B4) and (B5), we obtain the pre-FTA welfare levels of countries *S* and *L*:

$$W_S^* = \frac{(n+6)(\alpha - c)^2}{2(n+8)} + \frac{(\alpha - c)^2}{\beta(2n^2 + 4n + 3)^2}, \tag{B6}$$

$$W_L^* = \frac{(2n^2 + 4n + 1)(\alpha - c)^2}{2\beta(2n^2 + 4n + 3)} + \frac{n(\alpha - c)^2}{(n+8)^2} \tag{B7}$$

After the FTA is formed between countries *S* and *L*, the tariffs between the member countries are eliminated, $t_S = t_L = 0$. The FTA equilibrium sales in country *S*'s and *L*'s markets are given by

$$\begin{aligned} q_{SS}^F &= q_{SL}^F = \frac{\alpha - c}{n+2}, \quad q_{LS}^F = q_{LL}^F = \frac{\alpha - c}{\beta(n+2)}, \\ Q_S^F &= \frac{(n+1)(\alpha - c)}{n+2}, \quad Q_L^F = \frac{(n+1)(\alpha - c)}{\beta(n+2)} \end{aligned} \tag{B8}$$

Comparing Equations (B5) and (B8), the FTA decreases the domestic sales of firms *S* and *L* ($q_{ii}^F <$

$q_{ii}^{*},\ i = S, L$); however, it increases its exports ($q_{ji}^{F} > q_{ji}^{*},\ i, j = S, L,\ i \neq j$). It also increases the consumption of countries *S* and *L* ($Q_i^F > Q_i^*,\ i = S, L$).

Using Equation (B8), we obtain the FTA welfare levels of countries *S* and *L*:

$$W_S^F = \frac{(n^2 + 2n + 3)(\alpha - c)^2}{2(n+2)^2} + \frac{(\alpha - c)^2}{\beta(n+2)^2}, \tag{B9}$$

$$W_L^F = \frac{(n^2 + 4n + 1)(\alpha - c)^2}{2\beta(n+2)^2} + \frac{n(\alpha - c)^2}{(n+2)^2} \tag{B10}$$

Figure A1 shows a comparison between the pre-FTA and FTA welfare levels. The FTA improves (worsens) country *S*'s welfare in the region below (above) curve MN, *that is*, $W_S^F > W_S^*$ ($W_S^F < W_S^*$). The FTA benefits (hurts) country *L* in the region above (below) curve XY, *that is*, $W_L^F > W_L^*$ ($W_L^F < W_L^*$).

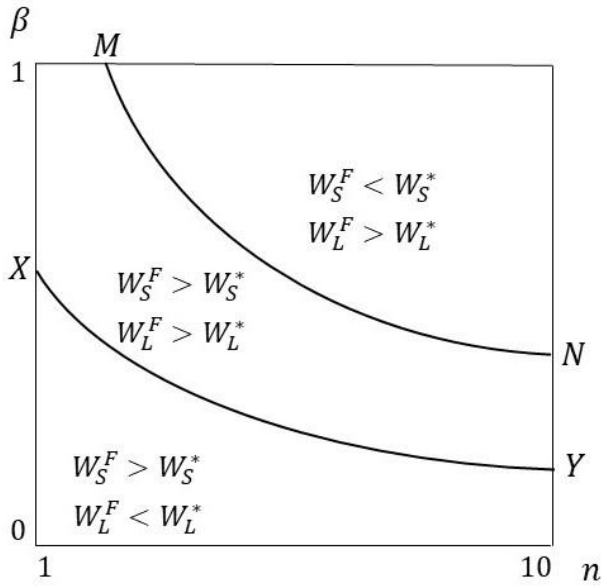

Figure A1. Comparison of pre-FTA and FTA welfare levels

Notes: $M = (2.1724, 1),\ N = (10, 0.39902),\ X = (1, 0.5625),\ Y = (10, 0.16333)$

With regard to the effect of an FTA on country *S*'s and *L*'s welfare levels in the model with

only import tariffs, we obtain the following proposition.

Proposition A1. *(i) The FTA between countries S and L is likely to improve country S's welfare the smaller the number of firms L, n, and/or the larger the market size of country L (the smaller the value of β). However, it will worsen country S's welfare if $n \geq 3$, and the market size of country L is slightly larger than that of country S (β is close to 1).*

(ii) The greater the number of firms L, and/or the closer the market size of country L to that of country S (the closer the value of β is to 1), the more likely it is that the FTA will make country L better off relative to the pre-FTA regime. However, it will make country L worse off if n is small enough and the market size of country L is significantly large (β is even less than 0.5625).

The intuition behind this proposition can be explained in the following manner.

(i) For country S's welfare

Following the FTA, country *S*'s consumer surplus increases, but its tariff revenue disappears. Firm *S*'s profits may or may not increase. The smaller the number of firms *L*, n, and/or the larger the market size of country *L* (the smaller the value of β), the more likely it is that the FTA will raise firm *S*'s profits through a significant increase in its export profits and, therefore, will increase country *S*'s welfare. However, if the values of n and β are quite large, the FTA will reduce firm *S*'s profits, and country *S*'s welfare.

(ii) For country L's welfare

The FTA enhances country L's consumer surplus but reduces its tariff revenue to zero. It may increase or decrease firm L's profits. The greater the number of firms L, n, and/or the closer the market size of country L to that of country S (the closer the value of β is to 1), the more likely it is that the FTA will increase firm L's profits, and hence, will benefit country L. However, if the values of n and β are small enough, the FTA will decrease firm L's profits because of a sharp decline in its local profits, and will hurt country L.

Appendix C. FTA *without* the harmonization of production subsidies

Consider an FTA without harmonizing the production subsidies. When countries S and L form an FTA, their governments remove tariffs on each other, $t_S = t_L = 0$, and choose a country-specific production subsidy that maximizes individual welfare.

Substituting $t_S = t_L = 0$ into Equations (3) and (4), and maximizing country i's welfare with respect to s_i, we obtain country i's reaction function under the FTA ($i = S, L$):

$$s_S = \frac{[(2n+1)\beta + n](\alpha - c) - n[(n-1)\beta + n]s_L}{(2n+1)\beta + 2(n+1)}, \quad (C1)$$

$$s_L = \frac{[(2-n)\beta + 3](\alpha - c) + [(n-2)\beta + n - 1]s_S}{n(4\beta + 3)} \quad (C2)$$

Solving Equations (C1) and (C2), we obtain the Nash equilibrium production subsidies under the FTA:

$$s_S^f = \frac{\beta(\alpha - c)}{\beta + 1}, \quad (C3)$$

$$s_L^f = \frac{\alpha - c}{n(\beta + 1)} \quad (C4)$$

From Equations (C3) and (C4), country-specific production subsidies under the FTA are positive.

Comparing Equations (12) with (C3), and (13) with (C4), we can show that countries *S* and *L* raise their production subsidies with the FTA, *that is*, $s_S^f > s_S^*$ and $s_L^f > s_L^*$. Moreover, using Equations (16), (C3), and (C4), we find that $s_S^f > s_C > s_L^f$ for $\beta > 1/n$, and $s_L^f > s_C > s_S^f$ for $\beta < 1/n$ ($s_S^f = s_L^f = s_C$ for $\beta = 1/n$).

Using Equations (C3) and (C4) yields the FTA equilibrium sales in country *S*'s and *L*'s markets:

$$q_{SS}^f = \frac{\beta(\alpha - c)}{\beta + 1}, \quad q_{SL}^f = \frac{\alpha - c}{n(\beta + 1)},$$
$$q_{LS}^f = \frac{\alpha - c}{\beta + 1}, \quad q_{LL}^f = \frac{\alpha - c}{n\beta(\beta + 1)},$$
$$Q_S^f = \alpha - c, \quad Q_L^f = \frac{\alpha - c}{\beta} \tag{C5}$$

From (C5), the FTA equilibrium prices in countries *S* and *L* are equal to the marginal cost: $p_S^f = p_L^f = c$.

Comparing Equation (A1) with (C5), we can show that an FTA without the harmonization of production subsidies may or may not reduce the domestic sales of firms *S* and *L*, but increases their exports ($q_{ji}^f > q_{ji}^*,\ i,j = S,L,\ i \neq j$). It also expands the consumption of countries *S* and *L* ($Q_i^f > Q_i^*,\ i = S,L$).

Using Equations (C3), (C4) and (C5), we obtain the FTA welfare levels of country *S* and *L*:

$$W_S^f = \frac{(\alpha - c)^2}{2}, \tag{C6}$$
$$W_L^f = \frac{(\alpha - c)^2}{2\beta} \tag{C7}$$

Equations (A5), (A6), (C6), and (C7) indicate that the welfare levels under an FTA without the

harmonization of production subsidies are the same as those under a deep FTA with the harmonization of production subsidies.